SUBJECT CODE: GE6253- ENGINEERING MECHANICS

YEAR/SEM: I/II

UNIT-1- BASICS AND STATICS OF PARTICLES

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COURSE OUTCOME:

CO1: The student will be able to study about the basics of forces and fundamental law of mechanics.

PROGRAM OUTCOME:

PO1: An ability to apply knowledge of computing, mathematics, science and engineering fundamentals appropriate to the mechanical engineering
PO2: An ability to analyze a problem, and identify and formulate the computing requirements appropriate to its solution.
PO3: An ability to design, implement, and evaluate a mechanical based system, process, component, or program to meet desired needs with appropriate consideration for public health and safety, cultural, societal and environmental considerations.
PO4: An ability to design and conduct experiments, as well as to analyze and interpret data.
PO5: An ability to use current techniques, skills, and modern tools necessary for computing practice.
PO6: An ability to analyze the local and global impact of computing on individuals, Organizations and society.
PO7: Knowledge of contemporary issues.
PO8: An understanding of professional, ethical, legal, security and social issues and responsibilities.
PO9: An ability to function effectively individually and on teams, including diverse and Multidisciplinary, to accomplish a common goal.
PO10: An ability to communicate effectively with a range of audiences.
PO11: Recognition of the need for and an ability to engage in continuing professional development.
PO12: An understanding of engineering and management principles and apply these to one’s own Work, as a member and leader in a team, to manage projects.
Introduction – Units and Dimensions

Engineering mechanics is that branch of science which deals with the behavior of a body when the body is at rest or in motion. It is mainly divided into statics and dynamics. The goal of this Engineering Mechanics course is to solve problems in mechanics as applied to plausibly real-world scenarios. Mechanics is the study of forces that act on bodies and the resultant motion that those bodies experience. With roots in physics and mathematics, Engineering Mechanics is the basis of all the mechanical sciences: civil engineering, materials science and engineering, mechanical engineering and aeronautical and aerospace engineering.

Laws of Mechanics
Lame’s theorem, Parallelogram and triangular Law of forces

The parallelogram of forces is a method to determine (or visualizing) the resultant of applying two forces to an object. It states, if two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point. Let P and Q be the force then resultant is R.

If there forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces. Suppose the three forces a, b and c are acting at a point and they are in equilibrium then equation is written as

\[ \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} \]

If two forces are acting simultaneously on a particle and can be represented by the two sides of a triangle taken in order, then the third side represents the resultant in the opposite order. Triangle Law of Vector Addition is a great method to add two vectors. Addition of three vectors using triangle Law and then the method is used to demonstrate the polygon law of vector addition.

Vectors- Vectorial representation of forces and moments- Vector operations: additions, subtraction, dot product, cross product

A quantity having direction as well as magnitude, especially as determining the position of one point in space relative to another. For example, displacement, velocity, acceleration, force, moment, and momentum. A quantity or phenomenon that exhibits magnitude only, with no specific direction, is called a scalar. Examples of scalars include speed, mass, electrical resistance, and hard-drive storage capacity.

"A unit vector is defined as a vector in any specified direction whose magnitude is unity i.e. 1. A unit vector only specifies the direction of a given vector. "A unit vector is denoted..."
by any small letter with a symbol of arrow hat. A unit vector can be determined by dividing the vector by its magnitude. A vector that indicates the position of a point in a coordinate system is referred to as position vector.

Suppose we have a fixed reference point O, then we can specify the position of a given point P with respect to point O by means of a vector having magnitude and direction represented by a directed line segment OP. This vector is called position vector.

**Coplanar Forces – Resolution and Composition of forces**

When several forces act on a body, then they are called a force system or system of forces. It is necessary to study the system of forces, to find out the net effort of forces on the body. Let us consider a wooden block resting on a smooth inclined plane. It is supported by a force P. the force system for this block consist of weight of the block, reaction of the block on the inclined plane and applied force.

Force system mainly divided into two;

1. Coplanar
   a. collinear  b. concurrent  c. parallel  d. non concurrent non parallel

2. Non-coplanar
   a. concurrent  b. parallel  c. non concurrent non parallel

**Equivalent systems of forces**

**Principle of transmissibility – Single equivalent force**

Steps to find equivalent system:

i. Replacing two forces acting at a point by their resultant.

ii. Resolving the force into two components.

iii. Cancelling two equal and opposite forces acting at a point.

iv. Attaching two equal and opposite forces at a point.

v. Transmitting a force along its line of action.

Principle of transmissibility states that “the state of rest or motion of a rigid body is unaltered if a force acting on a body is replaced by another force of same magnitude and direction, but acting anywhere on the body along the line of action of the force”. A couple is two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.
1. **What is engineering mechanics?**

Engineering mechanics is that branch of science which deals with the behavior of a body when the body is at rest or in motion. It is mainly divided into statics and dynamics. The goal of this Engineering Mechanics course is to solve problems in mechanics as applied to plausibly real-world scenarios. Mechanics is the study of forces that act on bodies and the resultant motion that those bodies experience. With roots in physics and mathematics, Engineering Mechanics is the basis of all the mechanical sciences: civil engineering, materials science and engineering, mechanical engineering and aeronautical and aerospace engineering.

2. **Write the application of engineering mechanics.**

Engineering mechanics has applications in many areas of engineering projects. To cite a few examples, it is applied in the design of spacecraft’s and rockets, analysis of structural stability and machine strength, vibrations, robotics, electrical machines, fluid flow and automatic control. As the oldest physical science, mechanics has very wide application in a variety of engineering fields. We human cannot stand without the Equilibrium; it is also applied to those inventions like cars, appliances, buildings, even in shoes and in a single paper.

3. **Define the statics.**

The branch of science which deals with the study of a body when body is at rest. Statics is the branch of mechanics that is concerned with the analysis of loads (force and torque, or "moment") on physical systems in static equilibrium, that is, in a state where the relative positions of subsystems do not vary over time, or where components and structures are at a constant velocity. When in static equilibrium, the system is either at rest, or its center of mass moves at constant velocity.

4. **Define dynamics.**

The branch of science which deals with the study of a body when body is in motion. Dynamics is a branch of physics (specifically classical mechanics) concerned with the study of forces and torques and their effect on motion, as opposed to kinematics, which studies the motion of objects without reference to its causes. Isaac Newton defined the fundamental physical laws which govern dynamics in physics, especially his second law of motion.

5. **What is kinematics?**
Kinematics is the branch of classical mechanics which describes the motion of points, bodies (objects) and systems of bodies (groups of objects) without consideration of the causes of motion. To describe motion, kinematics studies the trajectories of points, lines and other geometric objects and their differential properties such as velocity and acceleration. Kinematics is used in astrophysics to describe the motion of celestial bodies and systems, and in mechanical engineering, robotics and biomechanics to describe the motion of systems composed of joined parts (multi-link systems) such as an engine, a robotic arm or the skeleton of the human body.

6. **Write about kinetics.**

In physics and engineering, kinetics is a term for the branch of classical mechanics that is concerned with the relationship between the motion of bodies and its causes, namely forces and torques. Since the mid-20th century, the term "dynamics" (or "analytical dynamics") has largely superseded "kinetics" in physics text books; the term "kinetics" is still used in engineering. In mechanics, the Kinetics is deduced from Kinematics by the introduction of the concept of mass.

7. **Write the importance of engineering mechanics.**

The science of engineering mechanics is mainly concerned with the knowledge of state of rest or motion of bodies under the action of different forces. It is common experience that various bodies in the universe are either at rest or in motion. Naturally, to study the behavior of different bodies, the knowledge of engineering mechanics is a paramount importance, so as to execute the design and construction in the engineering field. Also, basic principles of mechanics are used in the study of subjects such as strength of materials, stability of structures, vibrations, fluid flow, electrical machines etc.

8. **What is mechanics of solids?**

Solid mechanics is the branch of continuum mechanics that studies the behavior of solid materials, especially their motion and deformation under the action of forces, temperature changes, phase changes, and other external or internal agents. Solid mechanics is fundamental for civil, aerospace, nuclear, and mechanical engineering, for geology, and for many branches of physics such as materials science. It has specific applications in many other areas, such as understanding the anatomy of living beings, and the design of dental prostheses and surgical implants.

9. **What is mechanics of fluids?**

Fluid mechanics is the branch of physics which involves the study of fluids (liquids, gases, and plasmas) and the forces on them. Fluid mechanics can be divided into fluid statics, the study of fluids at rest; and fluid dynamics, the study of the effect of forces on fluid motion. It is a branch of continuum mechanics, a subject which models matter without using the information that it is made out of atoms; that is, it models matter from a macroscopic viewpoint rather than from microscopic. Fluid mechanics, especially fluid dynamics, is an active field of research with many problems that are partly or wholly unsolved.

10. **Define rigid bodies.**
In physics, a rigid body is an idealization of a solid body in which deformation is neglected. In other words, the distance between any two given points of a rigid body remains constant in time regardless of external forces exerted on it. Even though such an object cannot physically exist due to relativity, objects can normally be assumed to be perfectly rigid if they are not moving near the speed of light.

11. Define deformable bodies.
Alteration in the shape or dimensions of an object as a result of the application of stress to it. Any alteration of shape or dimensions of a body caused by stresses, thermal expansion or contraction, chemical or metallurgical transformations, or shrinkage and expansions due to moisture change is called deformable body. In fact all the body will go for deformation when it is loaded. There is no rigid body in actual practice.

12. What is particle?
A particle is defined as a very small quantity of matter that may be considered to occupy a single point in space. Or, a particle is defined as a body whose dimensions are negligible and whose mass is concentrated at a point. Whether objects can be considered particles depends on the scale of the context; if an object's own size is small or negligible, or if geometrical properties and structure are irrelevant, then it can often be considered a particle. For example, grains of sand on a beach can be considered particles because the size of one grain of sand (~1 mm) is negligible compared to the beach, and the features of individual grains of sand are usually irrelevant to the problem at hand.

Laws of Mechanics Lame’s theorem,
Parallelogram and triangular Law of forces

It states, Everybody continues in a state of rest or uniform motion in a straight line unless it is compelled to change that state by some external force acting on it. When viewed in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by an external force. It is also defined as, if the net force (the vector sum of all forces acting on an object) is zero, then the velocity of the object is constant.

Newton's second law of motion - the rate of change of momentum is proportional to the imposed force and goes in the direction of the force. Newton's second law of motion can be formally stated as follows: The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object.

15. State Newton’s third law of motion.
For every action, there is an equal and opposite reaction. When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and
opposite in direction on the first body. This law is familiar in everyday situations; a force cannot be applied to an object unless something resists the reaction of that force. In order to walk across the floor, you must push back on the floor with your foot; then, according to Newton’s Third Law, the floor pushes forward on your foot, which propels you forward.

16. **State Parallelogram law of forces.**
   The parallelogram of forces is a method to determine (or visualizing) the resultant of applying two forces to an object. It states, if two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point. Let P and Q be the force then resultant is R.

17. **Write about sine law.**
   Let consider triangle with sides a, b, c and included angle α, β, γ then sine law is written as follows
   \[
   \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}
   \]
   In trigonometry, the law of sines, sine law, sine formula, or sine rule is an equation, relating the lengths of the sides of any shaped triangle to the sines of its angles.

18. **Write about cosine law.**
   Let consider triangle with sides a, b and c and included angle α, β, γ then cosine law is written as follows
   \[
   a^2 = b^2 + c^2 - 2bc \cos\alpha \\
   b^2 = a^2 + c^2 - 2ac \cos\beta \\
   c^2 = a^2 + b^2 - 2ab \cos\gamma
   \]

19. **State and briefly explain Lami’s theorem. (May/June 2012)**
   If there forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces. Suppose the three forces a, b and c are acting at a point and they are in equilibrium then equation is written as
   \[
   \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}
   \]

20. **Write about triangle law of forces.**
   If two forces are acting simultaneously on a particle and can be represented by the two sides of a triangle taken in order, then the third side represents the resultant in the opposite order. Triangle Law of Vector Addition is great method to add two vectors. Addition of three vectors using triangle Law and then the method is used to demonstrate the polygon law of vector addition.

21. **Write about of gravitational law of attraction.**
   Newton’s Universal Law of Gravitation states that any two objects exert a gravitational force of attraction on each other. The direction of the force is along the line joining the objects. The magnitude of the force is proportional to the product of the gravitational
masses of the objects, and inversely proportional to the square of the distance between them.

\[ F = G \frac{m_1 m_2}{r^2} \]

**Vectors- Vectorial representation of forces and moments - Vector operations:**
- additions, subtraction, dot product, cross product

22. **Define vectors.**
A quantity having direction as well as magnitude, especially as determining the position of one point in space relative to another. For example displacement, velocity, acceleration, force, moment and momentum. A quantity or phenomenon that exhibits magnitude only, with no specific direction, is called a scalar. Examples of scalars include speed, mass, electrical resistance, and hard-drive storage capacity.

23. **What is unit vector and position vector?**
"A unit vector is defined as a vector in any specified direction whose magnitude is unity i.e. 1. A unit vector only specifies the direction of a given vector." A unit vector is denoted by any small letter with a symbol of arrow hat. A unit vector can be determined by dividing the vector by its magnitude. A vector that indicates the position of a point in a coordinate system is referred to as position vector.

Suppose we have a fixed reference point O, then we can specify the position of a given point P with respect to point O by means of a vector having magnitude and direction represented by a directed line segment OP. This vector is called position vector.

24. **A force of magnitude 750N is directed along AB where A is (0.8,0,1.2)m and B is (1.4,1.2,0). write the vector form of the force. (Nov/Dec 2003)**

\[
BA = 0.6\hat{i} + 10\hat{j} - 12\hat{k}
\]

Unit vector = \[
\frac{0.6\hat{i} + 10\hat{j} - 12\hat{k}}{\sqrt{0.6^2 + 10^2 + 12^2}}
\]
= \[
0.33\hat{i} + 0.66\hat{j} - 0.66\hat{k}
\]

\[ F = 75(0.33\hat{i} + 0.66\hat{j} - 0.66\hat{k}) \]

\[ F = 25\hat{i} + 50\hat{j} - 50\hat{k} \]

25. **A force F = 10\hat{i} + 8\hat{j} - 5\hat{k} N acts at the point A is (2,5,6)m. what is the moment of the force about the point B (3,1,4)m? (Nov/Dec 2002)**

Solution:
\[
M_B = r \times F
\]
\[ r = (2 - 3)\hat{i} + (5 - 1)\hat{j} + (6 - 4)\hat{k} \]
\[ r = -\hat{i} + 4\hat{j} + 2\hat{k} \]
26. Two vector A and B are given. Determine their cross product and the unit vector along it. A = 2i + 3j + k and B = 3i - 3j + 4k. (Jan 2003)

Solution

\[ \vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \]

\[ \vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 3 & -3 & 4 \end{vmatrix} \]

\[ = 1\vec{i} - 5\vec{j} - 15\vec{k} \]

\[ |\vec{C}| = \sqrt{1^2 + (-5)^2 + (-15)^2} = 21.795 \]

Unit vector = \[ \frac{1\vec{i} - 5\vec{j} - 15\vec{k}}{21795} \]

\[ = \frac{1}{21795} \vec{i} - \frac{5}{21795} \vec{j} - \frac{15}{21795} \vec{k} \]

27. Find the unit vector of a factor \( \vec{F} = 3\vec{i} - 5\vec{j} + 8\vec{k} \)

Unit vector \( \lambda = \frac{\vec{F}}{|\vec{F}|} \)

\[ |\vec{F}| = \sqrt{3^2 + (-5)^2 + 8^2} = \sqrt{105} = 10.247 \]

\[ \lambda = \frac{3\vec{i} - 5\vec{j} + 8\vec{k}}{10.247} = 0.29\vec{i} - 0.49\vec{j} + 0.78\vec{k} \]

28. Define moment of forces.

A moment of force is the product of a force and its distance from an axis, which causes rotation about that axis. In principle, any physical quantity can be combined with a distance to produce a moment; commonly used quantities include forces, masses, and electric charge distributions. Unit of moment is N⋅m

Moment = Force x Perpendicular distance

29. Define couple.

Two parallel, non-collinear forces of equal magnitude having opposite sense are said to form a couple. Let us consider two forces \( \vec{F} \) and \( -\vec{F} \) of equal magnitude acting in opposite
directions whose lines of action are parallel and separated by a distance $d$. $d$ is the perpendicular distance between the lines of action of forces $F$ and $-F$. The sum of the components of these two forces in any direction is zero. The combined moment of these two forces about an axis perpendicular to the plane passing through any point like $O$ is not zero. In other words, the couple has tendency to rotate or turn the body.

30. **What is the effect of force and moment on a body?**
A force acting on an object may cause the object to change shape, to start moving, to stop moving, to accelerate or decelerate. When two objects interact with each other they exert a force on each other, the forces are equal in size but opposite in direction. A moment of force is the product of a force and its distance from an axis, which causes rotation of object about that axis. It may be clockwise or anti clockwise direction.

31. **Write the varignon’s theorem.**
Principle of moment states that the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point. The Varignon’s theorem states that the moment of a resultant of two concurrent forces about any point is equal to the algebraic sum of the moments of its components about the same point. It is used to solve the problems of beams.

32. **Write about resultant of two parallel forces.**
The forces acting on an object can be replaced with a single force that causes the object to behave in the same way as all the separate forces acting together did, this one overall force is called the resultant force.
If the resultant force acting on an object is ZERO then the object will remain stationary. If it was stationary when the resultant force became zero or move at a constant (steady) speed in a straight line if it was moving when the resultant force became zero.

33. **Find the magnitude of the resultant of the two concurrent forces of magnitude 60kN and 40kN with an included angle of 70° between them. (Apr/May 2011)**
Given:
P= 60kN
Q= 40kN
$\theta = 70^0$
To find:
Resultant, $R$
Solution:
By parallelogram law of forces,
$$R=\sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$
$$R=\sqrt{60^2 + 40^2 + 2\times60\times40 \cos 70}$$
$$R= 82.7 \text{ kN}$$

34. **Define resolution of a force.**
Resolution of forces means finding the components of a given force in two directions. Let force \( p \) makes \( \theta \) with X axis; it is required to find the components of the force \( P \) along X axis and Y axis. Component of \( P \) along X axis is \( P \cos \theta \) and along Y axis \( P \sin \theta \). That is single force can be resolved into two components - one directed upwards or downwards and the other directed rightwards or leftwards.

35. **Write the resolution of a force into a force and a couple.**

A given force \( F \) applied to a body at any point \( A \) can always be replaced by an equal force applied at another point \( B \) together with a couple which will be equivalent to the original force. In mechanics, a couple is a system of forces with a resultant (net or sum) moment but no resultant force. A better term is force couple or pure moment. Its effect is to create rotation without translation, or more generally without any acceleration of the centre of mass. In rigid body mechanics, force couples are free vectors, meaning their effects on a body are independent of the point of application. The resultant moment of a couple is called a torque.

**Coplanar Forces – Resolution and Composition of forces**

36. **What is meant by system of forces?**

When several forces act on a body, then they are called a force system or system of forces. It is necessary to study the system of forces, to find out the net effort of forces on the body. Let us consider a wooden block resting on a smooth inclined plane. It is supported by a force \( P \). the force system for this block consist of weight of the block, reaction of the block on the inclined plane and applied force.

37. **Write the classification of force system.**

Force system mainly divided into two;
1. Coplanar
   a. collinear  b. concurrent  c. parallel  d. non concurrent non parallel
2. Non-coplanar
   a. concurrent  b. parallel  c. non concurrent non parallel

38. **Define coplanar force. (May/June 2010)**

If all the forces in a system lie in a single plane then they are "coplanar forces."
There are different types of coplanar forces. They are:
   a. Coplanar Concurrent Forces.
   b. Coplanar Non-Concurrent Forces.
   c. Coplanar Parallel Forces.
   (i) Coplanar like Forces.
   (ii) Coplanar unlike Forces.

39. **What is coplanar collinear system of forces? (May/June 2010)**

If line of action of all forces acting in a single plane then they are "coplanarCollinear Force System". Let forces F1, F2 and F3 acting in a plane in the same line i.e., common line of
action then the system of forces is known as coplanar collinear force system. Hence in coplanar collinear system of forces, all the forces act in the same plane and a common line of action.

40. Define coplanar concurrent system of forces.
Line of action of all forces passes through a single point and forces lie in a single plane then they are “coplanar concurrent forces”. Let forces F1, F2 and F3 acting in a plane and these forces intersect or meet at a common point O. this system of forces is known as coplanar concurrent force system. Hence in a coplanar concurrent system of forces, all the forces act in the same plane and they intersect at a common point.

41. Write about coplanar parallel system of forces.
If all the forces are parallel to each other and lie in single plane then they are "coplanar parallel forces".

(i) Coplanar like Parallel Forces: All forces are parallel to each other and lie in a single plane and are action in the same direction.

(ii) Coplanar unlike parallel forces: All forces are parallel to each other and lie in single plane but acting in opposite direction.

42. Define coplanar non-concurrent non-parallel system of forces.
When the forces of their lines of action are not meet at a point and not parallel if the forces lie in the same plane, they are known as coplanar non concurrent non-parallel forces. This consists of a number of vectors that do not meet at a single point and none of them are parallel. These systems are essentially a jumble of forces and take considerable care to resolve.

43. What is non-coplanar force system? (Nov/Dec 2011)
All the forces do not lie in the single plane then the force system is called non-coplanar force system. All forces do not lie in a single plane and line of action does not pass through single point, then they are "non-coplanar non-concurrent forces". All forces do not lie in same plane but line of action passes through single point, and then they are "non-coplanar concurrent forces". All forces are parallel to each other but not lie in single plane then they are "non-coplanar parallel forces".

44. What are resultant forces?
The resultant of a force system is the Force which produces same effect as the combined forces of the force system would do. So if we replace all the combined forces of the force system would do. So if we replace all components of the force by the resultant force, then there will be no change in effect. Hence a single force which can replace a number of forces acting on a rigid body, without causing any change in the external effects on the body, is known as the resultant force.

Equilibrium of a particle - Forces in space – Equilibrium of a particle in space

45. Write about the principle of equilibrium. (Nov/Dec 2011)
The principle of equilibrium states that, a stationary body which is subjected to coplanar forces (concurrent or parallel) will be in equilibrium if the algebraic sum of all the external forces is zero and also the algebraic sum of moments of all the external forces about any point on or off the body is zero.

Mathematically it is expressed by the equations
\[ \sum F = 0 \]
\[ \sum M = 0 \]

46. Write the condition for equilibrium of non-concurrent forces system.
1. The algebraic, sums of the components of the forces along each of two lines at right angles to each other equal zero when forces are not intersecting at same point.
2. The algebraic sum of the moments of the forces about any origin equals zero for above force system.
\[ \sum F = 0 \]
\[ \sum M = 0 \]

47. Write the condition for equilibrium of coplanar concurrent forces system.
1. The algebraic, sums of the components of the forces along each of two lines at right angles to each other equal zero when forces are intersecting at same point.
2. The algebraic sum of the moments of the forces about any origin equals zero.
\[ \sum F = 0 \]
\[ \sum M = 0 \]

48. What is free body diagram?
Free body diagram of a body can be drawn by removal of all the supports (like wall, hinge, floor or any other body) and replace them by equal reactions which these supports applies on the body. The consideration of internal forces and external forces is essential part. A free body diagram, sometimes called a force diagram, is a pictorial device, often a rough working sketch, used by engineers and physicists to analyze the forces and moments acting on a body.

49. Write the steps involved in drawing a free body diagram. (Nov/Dec 2011)
1. Draw a diagram of the body completely isolated from all other bodies
2. Represent the action of each body or support that has been removed by suitable force.
3. Indicate known applied load by its magnitude and unknown load by a symbol.
4. Indicate the weight of the free body with a vertical downward arrow

50. Draw the free body diagram for given figure.
**51. Define stable equilibrium.**
A state in which a body tends to return to its original position after being disturbed, the body said to be stable equilibrium. This condition arises when some additional force sets up due to displacement which brings the body back to its original position. When the center of gravity of a body lies below point of suspension or support, the body is said to be in stable equilibrium. For example a book lying on a table is in stable equilibrium. A book lying on a horizontal surface is an example of stable equilibrium. If the book is lifted from one edge and then allowed to fall, it will come back to its original position.

**52. Define unstable equilibrium.**
If a body does not return back to its original position and moves farther apart after being slightly displaced from its rest position, the body is said to be in unstable equilibrium. This condition arises when the additional force causes the body to move apart from its rest position. A state of equilibrium of a body (as a pendulum standing directly upward from its point of support) such that when the body is slightly displaced it departs further from the original position.

**53. Define neutral equilibrium.**
When the center of gravity of a body lies at the point of suspension or support, the body is said to be in neutral equilibrium. Example: rolling ball. If a ball is pushed slightly to roll, it will neither come back to its original nor it will roll forward rather it will remain at rest. If the ball is rolled, its center of gravity is neither raised nor lowered. This means that its center of gravity is at the same height as before.

**54. What is the difference between a resultant force and equilibrium force? (May/June 2012)**
Resultant force: if a number of forces acting simultaneously on a particle, then these forces can be replaced by a single force which would produce the same effect as produced by all forces. This single force is called as resultant force. 
Equilibrant force: the force which brings the set of forces in equilibrium is known as equilibrant force. It is equal in magnitude and opposite in direction of the resultant force.

55. **What is resultant of concurrent forces in space?**

Resultant of a force system is a force or a couple that will have the same effect to the body, both in translation and rotation, if all the forces are removed and replaced by the resultant.
Let Rx, Ry and Rz are sum of force components in x, y and z directions then the resultant of this system is
\[
R = \sqrt{(Rx^2 + Ry^2 + Rz^2)}
\]

56. **Define equilibrium in space.**

A rigid body is said to be in equilibrium when the external forces (active and reactive too) acting on it forms a system equivalent to zero. A particle subjected to concurrent force system in space is said to be in equilibrium when the resultant force is zero. In other words,
\[
\vec{R} = R\hat{i} = Ry\hat{j} = Rz\hat{k} = 0
\]
Rx=0;  Ry=0;  Rz=0 hence the equations of equilibrium for a particle when subjected to concurrent forces in space can be written as
\[
\in Fx = 0 \in Fy = 0 \in Fz = 0
\]

**Equivalent systems of forces**

**Principle of transmissibility – Single equivalent force**

57. **What is an equivalent system of forces?**

An equivalent system for a given system of coplanar forces is a combination of a force passing through a given point and a moment about that point. The force is the resultant of all forces acting on the body. And the moment is the sum of all the moments about that point.
Hence the equivalent system consists of:
(i) A single force R passing through the given point P and 
(ii) A single moment MR

58. **What is meant by force couple system? (May/June 2013)**

When a number of forces and couples are acting on a body, they combined into a single force and couple having the same effect. This is called force couple system. It is also termed a equivalent systems. "The condition of equilibrium or motion of a rigid body is remaining unchanged, if force acting on the rigid body is replaced by another force of the same magnitude and same direction but, acting anywhere along the same line of action."

59. **Write the steps to find equivalent system.**

vi. Replacing two forces acting at a point by their resultant.
vii. Resolving the force into two components.
viii. Cancelling two equal and opposite forces acting at a point.
ix. Attaching two equal and opposite forces at a point.
x. Transmitting a force along its line of action.

60. **Define transmissibility of forces. (Nov/Dec 2008)**

It states that “the state of rest or motion of a rigid body is unaltered if a force acting on a body is replaced by another force of same magnitude and direction, but acting anywhere on the body along the line of action of the force”. A couple is two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

---

**Part-B**

1.(a). The two forces $P$ and $Q$ act on bolt A. Determine their resultant. (8)(May/June 2012)

![Diagram of forces P and Q acting on bolt A]
Solution:

(i) Law Of Cosine:

\[ R^2 = P^2 + Q^2 - 2PQ \cos B \]
\[ R^2 = [40^2 + 60^2 - (2 \times 40 \times 60 \cos 155^0)] \]
\[ R = 97.73 \text{N} \]

(ii) Law Of Sine :

\[ \frac{\sin A}{R} = \frac{\sin B}{Q} \]
\[ \frac{\sin A}{60} = \frac{\sin 155^0}{97.73} \]
\[ A = 15.04^0 \]
\[ \alpha = 20^0 + A \]
\[ \alpha = (20^0 + 15.04^0) \]
\[ \alpha = 35.04^0 \]

Result:
\[ R = 97.73 \text{N} \]
\[ \alpha = 35.04^0 \]

1.(b). A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 25 kN force directed along the axis of the barge, determine (i) the tension in each of the ropes, knowing that \( \alpha = 45^0 \), (ii) the value of \( \alpha \) such that the tension in rope 2 is maximum. (8) (May/June 2012)

Given:

Force exerted tug boats = 25KN
\[ \alpha = 45^0 \]

To Find:

(i) \( T_1 \times T_2 \)
(ii) \( \alpha \) White \( T_2 \) is minimum

Solution:

Using Sine Rule,
\[
\frac{T_1}{\sin 45^\circ} = \frac{T_2}{\sin 30^\circ} = \frac{25}{\sin 105^\circ}
\]

\[
\frac{T_1}{\sin 45^\circ} = \frac{25}{\sin 105^\circ} \Rightarrow T_1 = 18.30 \text{ KN}
\]

\[
\frac{T_2}{\sin 30^\circ} = \frac{25}{\sin 105^\circ} \Rightarrow T_2 = 12.94 \text{ KN}
\]

(ii) The minimum Value of \(T_2\) Occurs When \(T_1 \perp T_2\)

\[T_2 = 25 \times \sin 30^\circ \Rightarrow T_2 = 12.24 \text{ KN}\]

Corresponding value of \(T_1\) and \(\infty\) are,

\[T_1 = 25 \times \cos 30^\circ \Rightarrow T_1 = 21.7 \text{ KN}\]

\[\infty = (90^\circ - 30^\circ) = 60^\circ\]

2. Four forces act on bolt A as shown in figure. Determine the resultant of forces on the bolt. (16)

![Diagram of forces](image)

Given:

\[F_1 = 150 \text{ N} ; F_2 = 80 \text{ N} ; F_3 = 110 \text{ N} ;\]

To Find:

Resultant

<table>
<thead>
<tr>
<th>Force</th>
<th>Magnitude (N)</th>
<th>X-Component</th>
<th>Y-Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_1)</td>
<td>150</td>
<td>+129.9</td>
<td>75</td>
</tr>
<tr>
<td>(F_2)</td>
<td>80</td>
<td>-27.4</td>
<td>70.2</td>
</tr>
<tr>
<td>(F_3)</td>
<td>110</td>
<td>0</td>
<td>-110.0</td>
</tr>
</tbody>
</table>
\[
\begin{array}{c|c|c|c}
F_x & F_y & F_z \\
\hline
100 & 96.6 & -25.9 \\
\end{array}
\]

\[
R_x = 199.1 \\
R_y = 14.3
\]

\[R = (R_x \hat{i} + R_y \hat{j})
\]

\[R = (199.1 \hat{i} + 14.3 \hat{j})
\]

\[\tan \theta = \frac{R_y}{R_x} = \frac{14.3}{199.1}
\]

\[\Rightarrow \theta = 4.1^\circ
\]

\[R = \frac{14.3}{\sin 41^\circ} \Rightarrow R = 199.6 \text{N}
\]

3. A tower guy wire is anchored by means of bolt at A. The tension in the wire is 2500N. Determine (a) the components \(F_x, F_y, F_z\) of the force acting in the bolt, (b) the angles \(\theta_x, \theta_y, \theta_z\) defining the direction of force. (16)

Given:

\[F = 2500 \text{N} ; d_x = -40 \text{m} ; d_y = 80 \text{m} ; d_z = 30 \text{m}
\]

To Find:

(a) \(F_x, F_y, F_z\)

(b) \(\theta_x, \theta_y, \theta_z\)

Solution:

Position of A = (40, 0, -30)

B = (0, 80, 0)

\[AB = -(0-40)\hat{i} + (80-0)\hat{j} + (0-30)\hat{k}
\]
\[ AB = -40i + 80j + 30k \]
\[ AB = \frac{\sqrt{40^2 + 80^2 + 30^2}}{94.3m} \]

Total Distance from A to B
\[ AB = d = \sqrt{dx^2 + dy^2 + dz^2} \]
\[ AB = \frac{2500}{94.3} [-40i + 80j + 30k] \]
\[ F = 1060i + 2120j + 795k \]
\[ F_x = -1060N \]
\[ F_y = 2120N \]
\[ F_z = 795N \]

(b) Direction of Force:
\[ \cos \theta_x = \frac{F_x}{F} = \frac{-1060}{2500} \]
\[ \theta_x = 115.1^\circ \]
\[ \cos \theta_y = \frac{F_y}{F} = \frac{2120}{2500} \]
\[ \cos \theta_z = \frac{F_z}{F} = \frac{795}{2500} \]
\[ \theta_z = 71.5^\circ \]

4. A 200 Kg cylinder is hung by means of two cables AB and AC, which are attached to the top of the vertical wall. A horizontal force P perpendicular to the wall holds the cylinder in the position shown in figure. Determine the magnitude of P and the tension in each cable. (16) (Nov/Dec 2009)
Given:

A = (1.2, 0, 0)
B = (0, 10, 8)
C = (0, 10, -10)

To Find,

(i) Magnitude of P
(ii) Tension $T_{AB}$ & $T_{AC}$

Solution:

$P = P_i$
$W = -mg$
$W = -1962j$

$\lambda_{AC} = -0.0846_i + 0.705_j - 0.705_k$

$T_{AC} = T_{AC} \cdot \lambda_{AC}$

$T_{AC} = -0.0846T_{AC}j + 0.705T_{AC}j - 0.705T_{AC}k$

Equilibrium Condition,
\[ \sum F = 0 \\
(T_{AB} + T_{AC} + P + W) = 0 \\
(-0.0933T_{AB} - 0.0846T_{AC} + P) + (0.778T_{AB} + 0.705T_{AC} - 1962) + (0.622T_{AB} - 0.705T_{AC})k = 0 \\
\sum F_x = 0, \\
(-0.0933T_{AB} - 0.0846T_{AC} + P) = 0 \\
(0.778T_{AB} + 0.705T_{AC} - 1962) = 0 \\
(0.622T_{AB} - 0.705T_{AC}) = 0 \\
\]

Result:

P = 235N \\
T = 140N \& 1236N

5. A rectangular plate is supported by bracket at A and B and by a wire CD. Knowing that the tension in the wire is 200N, determine the moment about A of the force exerted by the wire on point C. (16) (Nov/Dec 2009)

\[ \begin{align*}
A &= (0, 0, -0.08) \\
B &= (0.3, 0, 0) \\
D &= (0, 0.24, -0.32) \\
\end{align*} \]

To Find:

Moment about A (M_A)
Solution:

\[ M_A = \frac{\gamma_C}{F} \times F \]

Where, \( \frac{\gamma_C}{F} \) is the Vector drawn from A to C

\[ \frac{\gamma_C}{F} = 0.3i + 0.08k \]

Unit Vector,

\[ \lambda = \frac{\overrightarrow{CD}}{CD} \]

\[ F = F \lambda = 200 \frac{\overrightarrow{CD}}{CD} \]

Resolving the Vector \( \overrightarrow{CB} \) into rectangular Component,

\[ \overrightarrow{CD} = -0.3i + 0.24j - 0.32k \]

\[ CD = 0.5m \]

\[ F = \frac{200}{0.5} [ -0.3i + 0.24j - 0.32k ] \]

\[ F = -120i + 96j - 128k \]

Substitute Values,

\[ M_A = \gamma_C \times F \]

\[ M_A = (0.3i + 0.08k) \times (-120i + 96j - 128k) \]

\[ M_A = -7.68i + 28.8j + 28.8k \]

6. A 4.8m beam is subjected to the force shown in figure. Reduce the given system of force to (a) an equivalent force couple system at A, (b) equivalent force couple system at B, (c) a single force or resultant.(16)
Solution:

a) Force Couple System at A

\[ R = \sum F \]
\[ R = (150 \mathbf{i} - 600 \mathbf{j} + 100 \mathbf{j} - 250 \mathbf{j}) \]
\[ R = -600 \mathbf{j} \downarrow \]
\[ M_A = \sum (\gamma \times F) \]
\[ M_A = [(1.6 \mathbf{i} \times (-600 \mathbf{j})) + (2.8 \mathbf{i} \times 100 \mathbf{j}) + (48 \mathbf{i} \times (-250 \mathbf{j}))] \]
\[ M_A = -1880 \text{Nm} \]

The Equivalent force Couple System at “A” is

\[ R = 600 \mathbf{N} \downarrow \]
\[ M_A = 1880 \text{Nm} \]

(b) Force Couple System at “B”

The Force R is Unchanged

\[ M_B = M_A + (BA \times R) \]
\[ M_B = -1880 \mathbf{R} + (-4.8 \mathbf{i} - 600 \mathbf{j}) \]
\[ M_B = 1000 \text{Nm} \]

The equivalent force Couple system at “B”

\[ R = 600 \mathbf{N} \downarrow \]
\[ M_A = 1000 \text{Nm} \]

(c) Single Force (or) Resultant:

\[ \gamma \times R = M_A \]
\[ \mathbf{i} \times (-600 \mathbf{j}) = -1880 \]
\[ -\mathbf{x} \times 600 = -1880 \]
\[ x = 3.13 \text{m} \]

Thus the single force equivalent to the given system is,
7. Four tugboats are used to bring an ocean liner to its pier. Each tugboat exerts 30 kN force in the direction shown in figure. Determine (a) the equivalent force couple system at the foremast O, (b) the point on the hull where single, more powerful tugboats should push to produce the same effect as the original four tugboats. (16) (May/June 2010)

Solution:

(a) Force Couple System at ‘O’

Resultant, \( R = \sum F \)
\[
\sum F = (15i - 26j) + (18i - 24j) + 80j + (2i + 21.2j)
\]
\[
\sum F = (54.2i - 58.8j)
\]

\[
M_O = \sum (\gamma \times F)
\]
\[
M_O = [-27i + 15j] + (15i - 26j) + (30i + 21j)x(12i - 24j) + (120i + 21j)x(-30j) + (90j - 21j)x(21.2i + 21.2j)]
\]
\[
M_O = -1868 KNm
\]

R = 80 KN

(b) Single tug Boat

\[
\gamma' = (x + 21j)
\]

\[
(\gamma' \times R) = M_o
\]
\[
(x + 21j) \times (54.2i - 58.8j) = -186.8k
\]
\[
-x(58.8)R - 1138k = -1868k
\]
\[
x = 12.41m
\]
8. Four forces of magnitude 10kN, 15kN, 20kN and 40kN are acting at a point O as shown in figure. The angle made by 10kN, 15kN, 20kN and 40kN with X-axis are 30°, 60°, 90° and 120° respectively. Find the magnitude and direction of the resultant force. (16) (Five times)

Given:

\[ R_1 = 10 \text{ kN}; \theta_1 = 30^\circ \]
\[ R_2 = 15 \text{ kN}; \theta_2 = 60^\circ \]
\[ R_3 = 20 \text{ kN}; \theta_3 = 90^\circ \]
\[ R_4 = 40 \text{ kN}; \theta_4 = 120^\circ \]

To find:

Resultant (R) and \( \theta \).

Solution:

X-axis,
\[ H = (R_1 \cos \theta_1 + R_2 \cos \theta_2 + R_3 \cos \theta_3 + R_4 \cos \theta_4) \]
\[ H = (10 \cos 30^\circ + 15 \cos 60^\circ + 20 \cos 90^\circ + 40 \cos 120^\circ) \]
\[ H = -8.84 \text{ kN} \]

Along Y-axis,
\[ V = R_1 \sin \theta_1 + R_2 \sin \theta_2 + R_3 \sin \theta_3 + R_4 \sin \theta_4 \]
\[ V = 10 \sin 30^\circ + 15 \sin 60^\circ + 20 \sin 90^\circ + 40 \sin 120^\circ \]
\[ V = 72.63 \text{ kN} \]

Magnitude of Resultant Force,
\[ R = \sqrt{H^2 + V^2} \]
\[ R = \sqrt{(-8.84)^2 + 72.63^2} \]
\[ R = 72.73 \text{ kN} \]
Direction of force (R)
\[ \theta = \frac{V}{H} \]
\[ \theta = 72.73^\circ \]
\[ \theta = -3.84^\circ \]
\[ \theta = 18.91^\circ \]

9. Three external forces are acting on a L-shaped body as shown in figure. Determine the equivalent system through point O. (16)

Given:
Force at A=2000N
B=1500N
C=1000N
Distance OA=200mm, OB=100mm & BC=200mm.

To Find:
Single resultant (R)
Single Moment (M)

Solution:
The force A is resolved into TWO,
Along X-axis = \(2000 \cos 30^\circ\) = 1732N
Along Y-axis = \(2000 \sin 30^\circ\) = 1000N

Resolving All Forces along X-axis,
\[ \sum F_x = (2000 \cos 30^\circ - 1500 - 1000) \]
\[ \sum F_x = -768N \]
\[ \sum F_y = (-2000 \sin 30^\circ) \]
\[ \sum F_y = -1000N \]

Resultant,
\[ R = \sqrt{\sum F_x^2 + \sum F_y^2} \]

\[ R = \sqrt{(-786)^2 + (-1000)^2} \]

\[ R = 1260.88 \text{N} \]

Moment of all forces about point ‘0’ is

\[ M_o = [(2000 \sin 30^\circ)X(200+1500)X(100)+(1000X300)] \]

\[ M_o = 250 \text{N.m} \]

10. Four forces 32kN, 24kN, 24kN and 120kN are concurrent at origin and are respectively directed through the points whose coordinates are A (2,1,6), B(4,-2,5), C(-3,-2,1), and D(5,1,-2). Determine the resultant of the system. (16) (May/June 2005)

Sol:

Let ‘O’ be the origin let \( F_1, F_2, F_3, \& F_4 \) be the Forces along \( OA, OB, OC \& OD \) Respectively,

Force along OA \( (F_1) \) = (magnitude of \( F_1 \)) x (unit vector along OA)

\[ = 38 \times \frac{2i+2j+6k}{\sqrt{2^2+1^2+6^2}} \]

\[ = 10i+5j+30k \]

Similarly, Force along OB \( (F_2) \)

\[ = 24 \times \frac{4i-2j+5k}{\sqrt{4^2+(-2)^2+5^2}} \]

\[ = 14.3i-7.156j+17.89k \]

Force, along OC \( (F_3) \) = 24 \times \frac{-3i-2j+k}{\sqrt{(-3)^2+(-2)^2+4^2}}

\[ = -19.25i-12.83j+6.417k \]

Force, along OD \( (F_4) \) = 120 \times \frac{5i+j-2k}{\sqrt{5^2+1^2+(-2)^2}}
\( = 109.55i + 21.91j - 43.82k \)

Resultant of the forces

\[ R = F_1 + F_2 + F_3 + F_4 \]
\[ R = (114.6i + 6.924j + 10.487k) \]

Magnitude of the resultant \( = \sqrt{(114.6^2 + 6.924^2 + 10.487^2)} \)
\[ = 115.297 \text{ KN} \]

\[ \theta_x = \cos^{-1}\left(\frac{119.61}{115.297}\right) \]
\[ \theta_x = 6.28^\circ \]

\[ \theta_y = \cos^{-1}\left(\frac{6.924}{115.297}\right) \]
\[ \theta_y = 86.56^\circ \]

\[ \theta_z = \cos^{-1}\left(\frac{10.487}{115.297}\right) \]
\[ \theta_z = 84.78^\circ \]

Resultant:

\[ R = 115.297 \text{ K} \]

11. Figure shows the coplanar system of forces acting on a flat plate. Determine (i) the resultant and (ii) x and y intercepts of the resultant. (16) (May/June 2010)

Given:

Force at A = 2045N
Angle with X-axis = 63.43°
Force at B=1805N
Angle with X-axis = 60°
Lengths, OA=4m,OB=3m,OC=2m & OD=3m

To Find:
(i) Resultant (R)
(ii) X & Y intercepts of resultant,

Solution:
(i) Force at A = 2240 N
   X-Component = (2240 X Cos 63.43°) = 1502.2 N
   Y-Component = (2240 X Sin 63.43°) = 2003.4 N

Force at B = 1805 N
   X-Component = (1805 X Cos 33.67°) = 1502.2 N
   Y-Component = (1805 X Sin 33.67°) = 1000.7 N

Force at C = 1500N
   X-Component = (1500 X Cos 60°) = 750 N
   Y-Component = (1500 X Sin 60°) = 1299 N

Force along X-axis,
\[ R_x = \sum F_x = [1001.9 + 1500 + (-750)] = 1250.3 N \]

Force along Y-axis,
\[ R_y = \sum F_y = (-2003.4 - 1000.7 + 1299) = -1705.1 N \]

Resultant force is given by,
\[ R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-1250.3)^2 + (-1705.1)^2} \]
\[ R = 2114.4 N \]

The angle made by the resultant with X-axis is given by,
\[ \tan \theta = \frac{R_y}{R_x} = \frac{-1705.1}{-1250.3} \]
\[ \theta = 53.7° \]

(ii) Intercepts of Resultant along X-axis & Y-axis,

Moment of R about 0 = Sum of moments of \( R_x = R_y \) at 0
\[ -2411.1 = (R_x X 0) + (R_y X x) \]
\[ X = \frac{-2411.1}{-1705.1} = 1.41 m \]

Moment of R about 0 = Sum of moments of \( R_x = R_y \) at 0
12. Two identical rollers, each of weight \( W = 1000N \), are supported by an inclined plane and a vertical wall as shown in figure. Find the reactions at the points of supports A, B and C. Assume all the surfaces to be smooth. (16) (Nov/Dec 2009)

Given:
- Weight of each roller = 1000N
- Radius of each roller same

To find:
- Reactions at points of supports A, B & C

Solution:

**Equilibrium of Roller (P):**

The resultant force in X and Y directions on roller “P” should be Zero.

\[
\begin{align*}
\sum F_x &= 0 \\
(R_y \sin 60^\circ - R_A \sin 30^\circ) &= 0 \\
R_y &= 0.577 R_A \\
\sum F_y &= 0 \\
(R_y \cos 60^\circ - R_A \cos 30^\circ - 1000) &= 0 \\
(0.577 R_A \cos 60^\circ + R_A \cos 30^\circ - 1000) &= 0 \\
R_A &= 866.17N
\end{align*}
\]

**Equilibrium of Roller (Q):**
\[ \sum F_x = 0 \\
(R_y \cdot \sin 30^\circ - R_y \cdot \sin 60^\circ - R_c) = 0 \\
(R_y \times 0.5 + 499.78 \cdot \sin 60^\circ - R_c) \]

\[ R_c = (0.5R_y + 499.78) \]

\[ \sum F_y = 0 \\
(R_y \cdot \cos 30^\circ - 1000 - R_y \cdot \cos 60^\circ) = 0 \\
(R_y \times \cos 30^\circ - 1000 - 499.78 \cdot \cos 60^\circ) = 0 \]

\[ R_y = 1443.3 \text{N} \]

\[ R_c = 1154.45 \text{N} \]

13. Two spheres, each of weight 1000N and of radius 25 cm rest in a horizontal channel of width 90 cm as shown in figure. Find the reactions on the points of contact A, B and C. (16) (Nov/Dec 2012)

**Given:**

Weight of Each sphere, \( W = 1000 \text{ N} \)

Radius, \( r = 25 \text{ cm} \)

Width of channel = 90 cm

**To Find:**

Reaction on the points of contact A, B and C.

**Solution:**

\[ AF = BF = FD = CE = 25 \text{ cm} \]

\[ EF = (25 + 25) = 50 \text{ cm} ; FG = 40 \text{ cm} \]

\[ \text{In } \Delta^o, EFG, EG = \sqrt{EF^2 - FG^2} = \sqrt{50^2 - 40^2} = 30 \text{ cm} \]
14. Two smooth circular cylinders, each of weight $W=1000\,N$ and radius 15 cm, are connected at their centres by a string AB of length=40 cm and rest upon a horizontal plane, supporting above them a third cylinder of weight $=2000\,N$ and radius 15cm as shown in figure. Find the force $S$ in the string AB and the pressure produced on the floor at the points of contact D and E. (16)
Solution:

\[ AC = (AH + FC) \]
\[ AC = (15 + 15) = 30 \text{cm} \]
\[ AH = \frac{1}{2} AB \]
\[ AH = \frac{1}{2} \times 40 = 20 \text{cm} \]

From \( \triangle ACH \),

\[ \sin \theta = \frac{AH}{AC} = \frac{20}{30} = 0.667 \]
\[ \theta = \sin^{-1}(0.667) \Rightarrow \theta = 41.836^\circ \]

Equilibrium of cylinder (3):

Resolving forces horizontally,
\[ (R_F \cdot \sin \theta - R_G \cdot \sin \theta) = 0 \]
\[ R_F = R_G \]

Resolving forces vertically,
\[ (R_F \cdot \cos \theta + R_G \cdot \cos \theta) = 2000 \]
\[ R_F = \frac{2000}{\cos 41.836^\circ} \]
\[ R_F = 1342.179 N \]

Equilibrium of cylinder (1):

\[ \sum F_x = 0 \]
\[ S - R_F \cdot \sin \theta = 0 \]
\[ S = (1342.179 \times \sin 41.836^\circ) \]
\[ S = 895.2 N \]
\[ \sum F_y = 0 \]
\[ (R_y - 1000 - R_x \cos \theta) = 0 \]
\[ R_y = 1000 + R_x \cos \theta \]
\[ R_y = (1000 + 1342.179 \times \cos 41.836^\circ) \]
\[ R_y = 2000 \text{N} \]

Equilibrium of cylinder 1, 2 & 3 taken together:

\[ (R_y + R_E - 1000 - 2000 - 1000) = 0 \]
\[ R_E = (4000 - R_y) \]
\[ R_E = 2000 \text{N} \]

15. Determine the magnitude and direction of force \( F \) shown in figure so that the particle \( A \) is in equilibrium. (16)

Solution:

For the particle \( A \) to be in Equilibrium \( \sum \vec{F} = 0 \), Must be satisfied.

\[ F_1 + F_2 + F_3 + F = 0 \]
\[ F_1 = 500 \hat{i}; F_2 = -900 \hat{j}; F_3 = F_3 \hat{AB} \]

Where,
\[ \lambda_{AB} = \frac{AB}{AB} \]
\[ \lambda_{AB} = \frac{-4i + 8j - 2k}{\sqrt{(-4)^2 + 8^2 + (-2)^2}} \]
\[ \lambda_{AB} = \frac{1}{9.17} (-4i + 8j - 2k) \]
\[ F_3 = \frac{800}{9.17} (-4i + 8j - 2k) \]
\[ F_3 = -345.96i + 697.92j - 174.48k \]
\[ F = F_xi + F_yj + F_zk \]
\[ = (500i - 900j - 348.96i + 697.92j - 174.48k + F_xi + F_yj + F_zk) = 0 \]

Equating the respective i, j, k components to zero,
\[ \sum_{i} F_x = 0 \Rightarrow 151.04 + F_x = 0 \Rightarrow F_x = -151.04N \]
\[ \sum_{j} F_y = 0 \Rightarrow -202.08 + F_y = 0 \Rightarrow F_y = 202.08N \]
\[ \sum_{k} F_z = 0 \Rightarrow -174.48 + F_z = 0 \Rightarrow F_z = 174.48N \]
\[ F = (-151.04i + 202.08j + 174.48k) \]
\[ F = \sqrt{(151.04)^2 + (202.08)^2 + (174.48)^2} \]
\[ F = 306.75N \]

Direction of force (F):
\[ \cos \theta_x = \frac{F_x}{F} = \frac{-151.04}{306.75} \]
\[ \theta_x = 119.5^\circ \]
\[ \cos \theta_y = \frac{F_y}{F} = \frac{202.08}{306.75} \]
\[ \theta_y = 48.8^\circ \]
\[ \cos \theta_z = \frac{F_z}{F} = \frac{174.48}{308.75} \]
\[ \theta_z = 55.3^\circ \]
16. Determine the magnitudes of forces $F_1$, $F_2$ and $F_3$ for equilibrium of the particle A shown in figure.

Solution:

A (0,0,0)
B (-1.83, 1.52, -1.22)
$F_1 + F_2 + F_3 + F_4 = 0$

$F_1 = F_1 \lambda_{AB} = \frac{AB}{AB}$

$AB = (\vec{x} + \vec{y} + \vec{z})$

$AB = -1.83\vec{i} + 1.52\vec{j} - 1.22\vec{k}$

$AB = \sqrt{(-1.83)^2 + (1.52)^2 + (-1.22)^2}$

$AB = 2.67m$

$\lambda_{AB} = [-0.69\vec{i} + 0.57\vec{j} - 0.46\vec{k}]$

$F_i = F_i[-0.69\vec{i} + 0.57\vec{j} - 0.46\vec{k}]$

$F_3 = F_3\vec{j}$

$F_3 = (F_3 \cos 40^\circ \vec{i} + F_3 \cos 50^\circ \vec{j} + F_3 \cos 60^\circ \vec{k})$

$F_3 = F_3(0.77\vec{i} + 0.64\vec{j} + 0.5\vec{k})$

$F_4 = -890\vec{j}$

$[F_1(-0.69i + 0.57j - 0.46k) + F_2j + F_3(0.77i + 0.64j + 0.5k) - 890j] = 0$

Equating the respective $i$, $j$ & $k$ components to zero,

$-0.69F_1 + 0.77F_3 = 0$

$0.57F_1 + F_2 + 0.64F_3 = 890$
17. Three cables are used to support the 10 kg cylinder shown in figure. Determine the force developed in each cable for equilibrium. (16) (Nov/Dec 2009)

Solution.

\[ \vec{F}_1 = \vec{F}_{1i} \hat{i} + \hat{j} + 2k \]
\[ \vec{F}_2 = -\hat{i} + 6\hat{j} - 3\hat{k} \]

\[ \vec{F}_3 = (-0.67\hat{i} + 0.67\hat{j} - 0.33k) \]
\[ \vec{F}_4 = (-0.6\hat{i} + 0.67\hat{j} - 0.33k) \]

Equation the respective \( i, j, k \) Components to Zero.

\[ (F_1 - 0.67F_4) = 0 \]
\(-98.1 + 0.67 F_3 = 0\)
\((F_3-0.33F_4) = 0\)

Solving above equations

\(F_1 = 98.1\text{N}\)
\(F_2 = 48.32\text{N}\)
\(F_3 = 146.42\text{N}\)

18. Compute the moment of the force \(P=2000\text{N}\) and of the force \(Q=1600\text{N}\) shown in figure about points A, B, C and D. (16)

Solution:

The inclination of force \((P)\) about the horizontal axis is \(\theta_1\)

\[\theta_1 = \tan^{-1}(\frac{0.9}{1.2})\]
\[\theta_1 = 36.9^\circ\]

The inclination of force \((Q)\) about the horizontal axis is \(\theta_2\)

\[\theta_2 = \tan^{-1}(\frac{0.9}{0.6})\]
\[\theta_2 = 56.3^\circ\]

Moment about “A”

Moment of \((P)\) about A,

\[M_A = [1.5i \times (160i+1200j)]\]
\[M_A = 1800\text{N.m}\]

Moment about “B”

Moment of force \((P)\) about B,

\[(M_B)_P = (-0.3i+0.9j) \times (1000i+1200j)\]
\[(M_B)_P = (-360-1440) = -1800\text{N.m}\]
\[(M_B)_P = 1800\text{N.m}\]

Moment of force \((Q)\) about “B”
\[ (M_B)_Q = [(-0.6i - 0.9j)X(888i - 1331j)] \]
\[ (M_B)_Q = (798 + 798) = 1598 Nm \]
\[ (M_B)_Q = 1598 Nm \]

Moment about (C):
Moment of force (P) about C,
\[ (M_C)_P = [1.8jX(1600i + 1800j)] = -2880 Nm \]
\[ (M_C)_P = 2880 Nm \]

Moment of force (Q) about “C”
\[ (M_C)_Q = [(-0.3i)X(888i - 1331j)] = 399 Nm \]
\[ (M_C)_Q = 399 Nm \]

Moment of force (P) about D,
\[ (M_D)_P = [(1.5i + 1.8j)X(1600i + 1200j)] \]
\[ (M_D)_P = (1800 - 2880) = -1080 Nm \]

Moment of force (Q) about “D”
\[ (M_D)_Q = [1.2iX(888i - 1331j)] = 1597 Nm \]
\[ (M_D)_Q = 1597 Nm \]

19. Determine the magnitude and direction of the resultant of the forces acting on the hook shown in figure. (16)
Solution:
Applying equilibrium condition,
\[ \sum F_x = 0 \]
\[ (200\cos 30^\circ + 250\cos 35^\circ - 90\cos 65^\circ) = 354.69 \]
\[ \sum F_y = 0 \]
\[ (250\sin 35^\circ + 90\sin 65^\circ - 200\sin 30^\circ - 100) = 56.55 \]
\[ R = \sqrt{\sum F_y^2 + \sum F_x^2} \]
\[ R = \sqrt{56.55^2 + 354.69^2} \]
\[ R = 359.17 N \]
\[ \tan \theta = \frac{\sum F_y}{\sum F_x} \]
\[ \tan \theta = \frac{56.55}{359.17} \]
\[ \theta = \tan^{-1} \left( \frac{56.55}{359.17} \right) \]
\[ \theta = 8^\circ \]

20. Determine the tension in cables AB and AC required to hold the 40kg crate show in figure. (16) (May/June 2004)

Solution:
Applying equilibrium condition,
\[ \sum F_x = 0 \\
(450 - T_c \cos 30^\circ - T_B \cos 35^\circ) = 0 \\
(T_c \cos 30^\circ + T_B \cos 50^\circ) = 450 \\
\sum F_y = 0 \\
(T_c \sin 30^\circ + T_B \sin 50^\circ) = 392.4 \\
T_c = \frac{392.4 - T_B \sin 50^\circ}{\sin 30^\circ} \]

From equations (1) & (2)

\[ 450 = (\frac{392.4 - T_B \sin 50^\circ}{\sin 30^\circ}) \cos 30^\circ + T_B \cos 50^\circ \]

\[ T_B = 333.79 \text{N} \]

From Equations (1),

\[ T_C \cos 30^\circ + (333.79 \times \cos 50^\circ) = 450 \]

\[ T_C = 270.72 \text{N} \]